

Toward a precise definition of limit

Assignment 2: Meeting a specified tolerance

1. Consider the function $f(x) = 4x$ for x near $a = 3$.
 - (a) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 1 of 12. That is, $f(x)$ must be between 11 and 13.
 - (b) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.5 of 12.
 - (c) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.2 of 12.
 - (d) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within ϵ of 12.
Note 1: The symbol ϵ is a lower case Greek letter called “epsilon.”
Note 2: In this part, the tolerance is a variable. You should expect your result to depend on ϵ .

2. Consider the function $f(x) = x^2$ for x near $a = 3$.
 - (a) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 1 of 9. That is, $f(x)$ must be between 8 and 10.
 - (b) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.5 of 9.
 - (c) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.2 of 9.
 - (d) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within ϵ of 9.